Iowa Statewide Assessment of Student Progress (ISASP)

Mathematics Test Specifications, Grades 3-11

Table of Contents

Mathematics Test Specifications

Introduction

The Iowa Statewide Assessment of Student Progress (ISASP) includes individual assessments in English Language Arts (ELA), Mathematics, and Science intended for use within the last 12 weeks of the academic year. These summative assessments measure student achievement, growth and college and career readiness based on the Iowa Core Standards.

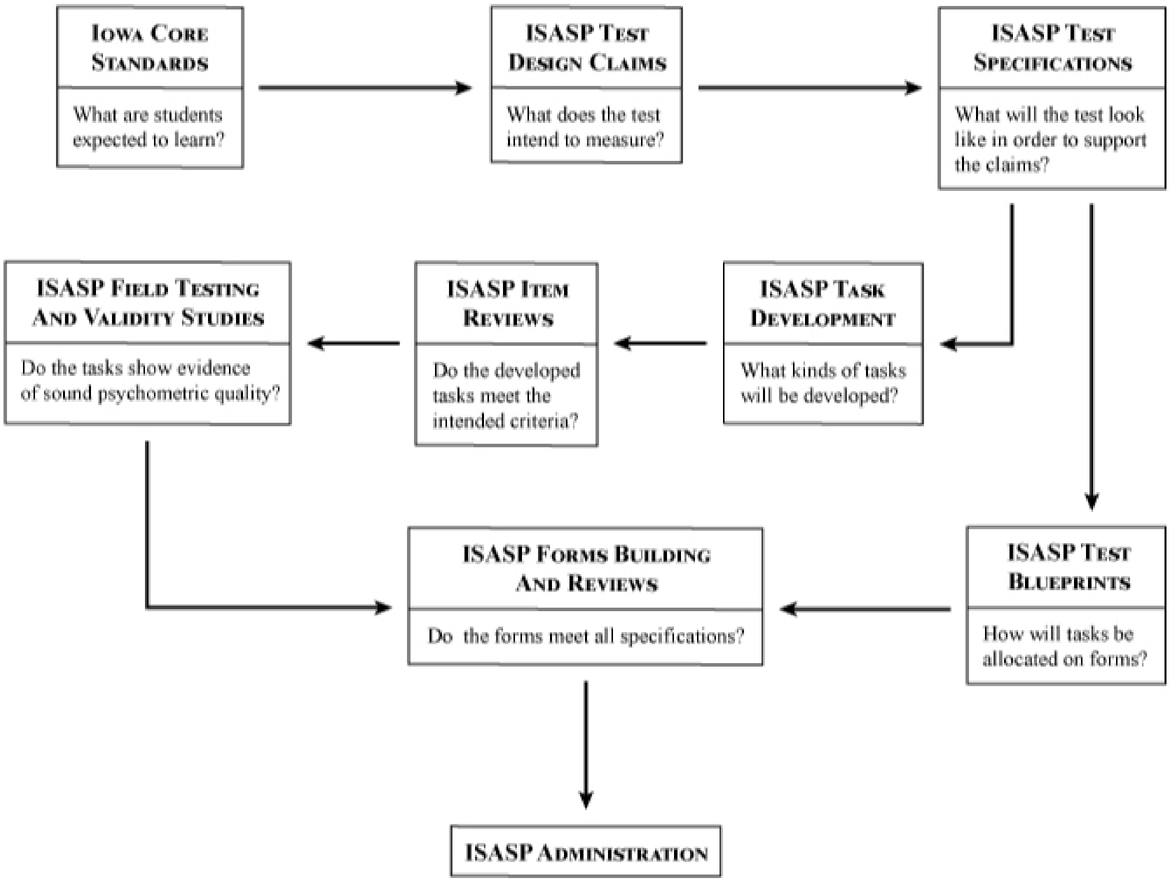
The ISASP Mathematics tests emphasize understanding, discovery, and quantitative thinking in mathematics. As a result, these tests provide educators, parents, and students with meaningful information.

The scope and sequence of the mathematics tests have been defined by the Iowa Core Standards in Mathematics. This document presents test specifications of the ISASP Mathematics for grades 3-11. Specifically, it has three main sections: 1) test design claims, 2) test and design specifications, and 3) enhanced blueprints.

Evidence-Centered Design and the Iowa Statewide Assessment of Student Progress

Evidence-Centered Design (ECD) presents a rigorous framework for building assessments to ensure "that the way in which evidence is gathered and interpreted [during the test development process] is consistent with the underlying knowledge and purposes the assessment is intended to address." (Mislevy, Almond, and Lukas, 2003, p.2). While ECD in the largest sense encompasses all aspects of the assessment program, it has particular significance for test development. ECD provides test developers with a means for decision making and documenting essential validity evidence around the claims the assessment is intending to measure, the selection and development of the specific tasks that will be given to elicit student responses, and the creation of the test specifications to be followed in forms assembly. Figure 1 provides a conceptual overview of the various stages of an evidence-based process.

Figure 1. An evidence-based approach to building the ISASP



Following ECD principles results in the development of ISASP assessments that can be mapped back directly to claims based on the Iowa Core Standards. As Herman and Linn (2015, p. 6) noted, "The transparency of the various ECD stages also provides a means for trying to assure that an assessment will represent the depth and breadth of standards and claims it is intended to measure. Each stage influences and constrains subsequent ones…". These stages work together to produce assessments that elicit the intended evidence to support the claims made by the ISASP.

Test Design Claims

The Mathematics assessments of ISASP has been designed and developed to support the following claims with respect to student performance:

- Students demonstrate progress toward college and career readiness in Mathematics.
- Students demonstrate growth from grade to grade in Mathematics.

Students will demonstrate their understanding of the Iowa Core Standards in Mathematics. Content related claims for grades 3 - 11 are given in Tables 1-7.

Table 1. Grade 3 – Content Related Claims

Grade 3	Claims
Operations and Algebraic Thinking	Students can represent and solve problems involving the four operations (multiplication and division within 100), understand the relationship between multiplication and division, and identify patterns in arithmetic.
Number and Operations in Base Ten	Students can use place value and properties of operations to perform multi-digit arithmetic.
Number and Operations – Fractions	Students can demonstrate a developing understanding of fractions as numbers.
Measurement and Data	Students can solve problems involving measurement and estimation, represent and interpret data, understand concepts of area and perimeter.
Geometry	Students can reason with shapes and their attributes.

Table 2. Grade 4 – Content Related Claims

Grade 4	Claims
Operations and Algebraic Thinking	Students can use the four operations with whole numbers to solve problems, demonstrate familiarity with factors and multiples, and analyze patterns.
Number and Operations in Base Ten	Students can generalize place value understanding and use it to perform multi-digit arithmetic.
Number and Operations – Fractions	Students can demonstrate understanding of fraction equivalence and ordering and decimal notation for fractions, build fractions from unit fractions, and compare decimal fractions.
Measurement and Data	Students can solve problems involving measurement, convert units from larger to smaller, represent and interpret data, and understand concepts of angles.
Geometry	Students can demonstrate understanding of lines and angles and classify shapes by properties of their lines and angles.

Table 3. Grade 5 – Content Related Claims

Grade 5	Claims
Operations and Algebraic Thinking	Students can interpret numerical patterns and analyze patterns and relationships.
Number and Operations in Base Ten	Students can demonstrate understanding of the place value system and perform operations with multi-digit whole numbers and decimals to hundredths.
Number and Operations – Fractions	Students can use equivalent fractions to add and subtract fractions and apply previous understanding to multiplying and dividing fractions.
Measurement and Data	Students can convert like measurement units, represent and interpret data, demonstrate understanding of the concept of volume and relate it to multiplication and to addition.
Geometry	Students can solve problems in the coordinate plane and classify two-dimensional figures based on their properties.

Table 4. Grade 6 – Content Related Claims

Grade 6	Claims
Ratios and Proportional Relationships	Students can demonstrate understanding of ratio concepts and use ratio reasoning to solve problems.
The Number System	Students can divide fractions by fractions, compute with multi-digit numbers, find factors and multiples, and demonstrate understanding of rational numbers.
Expressions and Equations	Students can apply understanding of arithmetic to algebraic expressions, solve one-variable equations and inequalities, and analyze relationships between variables.
Statistics and Probability	Students can demonstrate a developing understanding of statistical variability and can summarize/describe distributions.
Geometry	Students can solve problems involving area, surface area, and volume.

Table 5. Grade 7 – Content Related Claims

Grade 7	Claims
Ratios and Proportional Relationships	Students can demonstrate understanding of proportional relationships and use them to solve problems.
The Number System	Students can use knowledge of fractions to perform the four operations with rational numbers.
Expressions and Equations	Students can demonstrate understanding of equivalent expressions and can solve problems using numerical and algebraic expressions.
Statistics and Probability	Students can use random sampling to draw inferences, draw informal comparative inferences, and evaluate probability models.
Geometry	Students can describe geometrical figures and relationships between them and solve problems involving angle measure, area, surface area, and volume.

Table 6. Grade 8 – Content Related Claims

Grade 8	Claims
The Number System	Students can demonstrate understanding irrational numbers and how to approximate them by rational numbers.
Expressions and Equations	Students can demonstrate understanding of radicals, integer exponents, lines and linear equations, and solutions to linear equations and pairs of linear equations.
Functions	Students can compare functions and demonstrate understanding of their use in modeling relationships between quantities.
Geometry	Students can demonstrate understanding of congruence and similarity, apply the Pythagorean Theorem, and solve problems involving volume of cylinders, cones, and spheres.
Statistics and Probability	Students can demonstrate an understanding of patterns of association in bivariate data.

Table 7. Grades 9-11 – Content Related Claims

Grades 9-11	Claims
Number and Quantity	Students can extend the properties of exponents to rational exponents, reason quantitatively and use units to solve problems, and perform operations on matrices.
Algebra	Students can interpret the structure of expressions, perform arithmetic operations on polynomials, create equations that describe numbers or relationships, and represent and solve inequalities.
Functions	Students can use function notation, compare functions, and demonstrate understanding of their use in modeling relationships between quantities.
Geometry	Students can demonstrate understanding of congruence and similarity; prove geometric theorems; solve problems involving volume of cylinders, cones and spheres; and apply geometric concepts in modeling situations.
Statistics and Probability	Students can interpret categorical and quantitative data, interpret linear models, and understand independence and conditional probability and use them to interpret data.

Test and Design Specifications

Test and design specifications provide guidelines for developing sound and aligned assessments to support the claims of the assessment. The test specifications presented in this document reflect the depth and breadth of the performance expectations of the Iowa Core Standards. The test specifications include critical information about the domains of the Iowa Core to be assessed, the types of items to be used, the cognitive complexity and breadth of the items, and the statistical targets.

Domains Assessed

Tables 8 and 9 provide the domains and domain coverage targets in the Iowa Core that are assessed and reported for the Mathematics tests of the ISASP at Grades 3–11. For each Iowa Core domain, the content-related claim referenced in the previous section is made based on student performance.

Table 8. Iowa Core Math Domains Assessed – Grades 3-8

Iowa Core Math Domains	Grade 3	Grade 4	Grade 5	Grade 6	Grade 7	Grade 8
Operations and Algebraic Thinking	31-37%	16-22%	13-18%			
Number and Operations in Base Ten	14-20%	19-24%	23-28%			
Number and Operations – Fractions	11-14%	22-27%	23-28%			
Measurement and Data	23-29%	19-24%	18-23%			
Geometry	11-14%	11-16%	13-18%	12-17%	18-22%	19-23%
Ratios and Proportional Relationships				14-19%	18-22%	
The Number System				21-26%	20-24%	9-13%
Expressions and Equations				29-33%	22-27%	28-32%
Statistics and Probability				12-17%	11-16%	15-19%
Functions						20-24%

Table 9. Iowa Core Math Domains Assessed – Grades 9-11

Iowa Core Math Domains	Grade 9	Grade 10	Grade 11
Geometry	11-17%	26-31%	17-23%
Statistics and Probability	11-17%	11-17%	11-17%
Functions	17-23%	14-20%	20-26%
Algebra	29-34%	17-23%	20-26%
Number and Quantity	17-23%	17-23%	17-23%

Item Types

Measuring the depth and breadth of the current Iowa Core Standards requires a balanced and layered approach that incorporates a range of tasks and stimulus materials. Considerations while creating these materials include providing a variety of realistic contexts for math, integrating situations and prompts from curricular areas such as science and social studies; measuring key problem-solving skills in areas of number sense and operations, geometry, algebraic patterns and statistics/probability; and minimizing the amount of reading necessary to ensure that only mathematical skills are truly being measured.

Selected-response items are excellent for efficiently evaluating student knowledge and understanding of a variety of concepts and content included within the Iowa Core. Additional assessment formats measure those skills that are not easily assessed by these more traditional formats. Multiple types of item formats in the assessments expands and improves the measurement of student understanding and proficiency overall. The rigor of the current Iowa Core Mathematics Standards is mirrored by employing a robust suite of traditional and nontraditional item types, including:

- **Technology-enhanced items (TEIs):** These online items require students to engage in tasks designed to use complex thought processes. These items take advantage of the latest computer-based technologies. They may include response interfaces such as hot spots, drag-and-drop, point-and-click, cloze, and graphing; or require students to provide or select multiple responses to a single question. TEIs in mathematics can use online tools such as equation editors to take advantage of automated scoring. All TEIs are machine scored. Some specific examples of technology-enhanced item types are included in Table 10.
- **Selected-response items:** These items are efficient to administer and offer strong technical properties. These items can be written to address varying levels of cognitive complexity to measure students' skills and knowledge at three cognitive levels. All selected-response mathematics items at grades 3-8 have four options; all selected-response mathematics items at grade 9-11 have five options.
- **Two-part items:** These items require students to engage in each of two parts based on a common stimulus. Each part is worth one point. Students receive two points for answering both parts correctly; they may also receive one point for a partially correct answer.

The number of items per item types for grades 3-11 is given in Table 11.

Table 10. Examples of TEI Types

Item Type	Description
Drop-down Item	This item type allows students to make a selection from a drop-down menu.
Fill-in Item	This item type allows students to type in a text-based response using a keyboard (virtual or physical).
Open-ended Item	This item type allows students to type in a text-based response using a keyboard (virtual or physical).
Order Item	This item type allows students to order options into a sequence.
Hot Spot Item	This item type allows students to select one or more regions on a graphic or image to identify their choice.
Graphing Item	This item types allows students to manipulate, create, or edit, line graphs, scatterplots, function graphs, pie charts, and bar graphs.
Manipulative Item	This item type allows students to work with interactives such as animation, simulation, probability spinner, or other component.

Table 11. ISASP Mathematics Test – Number and types of operational items

Grade	Selected-Response Items (1 point)	Technology-Enhanced Items (1 point)	Two-Part Items (2 points)	Total Items
3	35-37	3-5	1	41
4	37-39	3-5	1	43
5	40-42	3-5	1	46
6	42-44	3-5	1	48
7	45-47	3-5	1	51
8	47-49	3-5	1	53
9	35-37	3-5	1	41
10	35-37	3-5	1	41
11	35-37	3-5	1	41

Cognitive Complexity

The depth-of-knowledge (DOK) should be consistent between what is required by the Iowa Core Standards and the items on the ISASP. To ensure this consistency, all items have been reviewed for their cognitive demand to ensure that what students are expected to know and do is consistent between the two and that the item-level DOKs meet or exceed the DOK levels specified for each standard in the Iowa Core. Table 12 describes these levels. The result is a full range of item complexity, where each item on the ISASP has been assigned one of three Cognitive Level descriptors. Table 13 summarizes percentage of items per DOK level

Table 12. ISASP Cognitive Descriptions

Cognitive Level	Description
Essential Competencies (DOK 1)	This level of difficulty involves recalling information such as facts, definitions, terms, or simple one-step procedures.
Conceptual Understanding (DOK 2)	This level of difficulty requires engaging in some cognitive processing beyond recalling or reproducing a response. A conceptual understanding item requires students to make some decisions as to how to approach the problem or activity and may require them to employ more than a single step.
Extended Reasoning (DOK 3)	This level of difficulty requires problem solving, planning, and/or using evidence. These items require students to develop a strategy to connect and relate ideas in order to solve the problem, and the problem may require that the student use multiple steps and draw upon a variety of skills.

Table 13. Percentage of items per DOK level for ISASP Mathematics test

DOK Level	Grade 3	Grade 4	Grade 5	Grade 6-8	Grade 9-11
DOK 1	20-35%	20-35%	20-35%	20-35%	20-35%
DOK 2	45-60%	45-60%	45-60%	45-65%	45-65%
DOK 3	10-25%	10-25%	10-25%	10-25%	10-25%

Statistical Specifications

To ensure support for claims that make inferences about student achievement, growth and readiness, both classical and IRT-based statistics are used to assemble test forms. For classical statistics the selection of items will be limited to those that have p-values within an acceptable range (0.20 to 0.90) and discrimination indices greater than 0.20. For IRT estimates, the selection of items will be based on a-parameters that are above 0.4 and b-parameters between -3.0 and 3.0.

Test Blueprints

The assessments in mathematics are rigorous, assessing what students can do with what they have learned. Items included in the assessment are carefully selected from the full range of content of the Iowa Core, and require a range of cognitive skills. Students demonstrate their understanding of concepts and procedures, solving problems, analyzing data, and communicating results.

For Grades 3-8, students must demonstrate an understanding of mathematics concepts, relationships, visual representations, and problem solving. Questions deal with number sense and operations, algebraic patterns and connections, data analysis, probability, statistics, geometry, and measurement. A variety of visual stimuli are used to present quantitative information or geometric representations for problem-solving activities. Possible student responses are selected to engage students in the relevant mathematical processes related to each question and to identify common misconceptions in students' mathematical thinking.

The primary objective of the Grades 9-11 tests is to measure students' ability to engage in appropriate mathematical reasoning and to solve quantitative problems. The questions present problems that require estimation, data interpretation, and sound thinking. Questions are drawn from the areas of number sense and operations, algebraic patterns and connections, data analysis, probability, statistics, geometry, and measurement. Calculations are kept to a minimum. In addition, some questions require students to analyze a problem and select the sequence of mathematical steps, or "set-up," that would yield the correct solution.

Tables 8 and 9 provide a summary of the percentage of items within total test by grade level by domain. More detailed test blueprints are given in the "Enhanced Blueprints" section.

Alignment Evidence

Alignment to the Iowa Core Standards has been a guiding principle of the development of the ISASP. Since the Iowa Core Standards were adopted by the state in July 2010, these standards define and shape the development and research necessary to build assessments aligned to the Iowa Core Mathematics.

To produce items that are aligned, Iowa Testing Programs follows a well-defined development process that helps to ensure the appropriate balance and representation of content. This process includes the following steps:

- Creation of test claims for the assessment that tie directly to the Iowa Core Standards in Science.
- Creation of test specifications that define the domains, standards and cognitive processes to be measured
- Development of test materials by Iowa educators and content experts that are aligned to the Iowa Core Standards
- Verification of these alignments by focus groups of Iowa educators who are actively teaching English Language Arts, Mathematics, and Science at the appropriate grade levels
- Continued evaluation of the items throughout field testing to confirm the items measure the standards as originally intended

Calling on the expertise of Iowa educators from the very beginning of the development process, including the initial conceptualization of the materials, is a defining feature of the ability to demonstrate alignment to the Iowa Core Standards.

Forms Construction

Forms construction for ISASP assessments in all subject areas begins by selecting the operational items for an administration. Using the items available in the item pool, content specialists and measurement experts from Iowa Testing Programs (ITP) assemble new forms by selecting items that meet the Enhanced Blueprints and targeted statistical specifications described previously in this document. The test assembly process is an iterative one, involving ITP development and measurement experts and teams of Iowa teachers with classroom experience who participate in item and test review panels.

Measurement and content experts review the newly assembled tests to ensure specifications and difficulty levels have been maintained. Although every item on the test has been previously scrutinized by Iowa educators and curriculum experts for alignment to Iowa Core, these attributes are reexamined for each item selected for the assembled test. Items are also further examined for their statistical quality, range of difficulties, and information and contributions given the selection of other items for the test.

Measurement and content experts also review each assembled form to ensure a wide variety of content is represented in the test items, to verify that the test measures a broad sampling of student skills within the content standards, and to eliminate “cueing” of an answer based on the content of another item appearing in the test. Additional reviews are designed to verify that keyed answer choices are the only correct answer to an item and that the order of answer choices on the test form varies appropriately. If any of these procedures uncovers an unsatisfactory item, the item is replaced with an item from the item bank and the review process begins again. This process for reviewing each newly constructed test forms helps to ensure each test will be of the highest possible quality.

The match between the targeted number of items per domain and the number of items found on the ISASP forms in 2019, 2021 and 2022 is provided in the Appendix (Tables 14 to 22). These tables provide evidence that any areas in need of improvement identified in the 2018 HumRRO ISASP Alignment Report (Dickinson, Michaels, & Thacker, 2018) have been incorporated into subsequent forms. Specifically, this includes the Mathematics domain Geometry in grade 3 (Table 14).

Mathematics Enhanced Blueprints – Grade Level Tables

Grade 3

Grade	Domain	Cluster	Standard #	Description	Range of operational items	Approx % of total test
3	**NBT**			**Number & Operations in Base Ten**	**5-7**	**14-20%**
3	NBT	A	0	**Use place value understanding and properties of operations to perform multi-digit arithmetic.**	**5-7**	**14-20%**
3	NBT	A	1	Use place value understanding to round whole numbers to the nearest 10 or 100.	1-3	3-9%
3	NBT	A	2	Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of operations, and/or the relationship between addition and subtraction.	1-4	3-11%
3	NBT	A	3	Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9 x 80, 5 x 60) using strategies based on place value and properties of operations.	1-4	3-11%
3	**NF**			**Number & Operations - Fractions**	**4-5**	**11-14%**
3	NF	A	0	**Develop understanding of fractions as numbers. (Grade 3 expectations in this domain are limited to fractions with denominators 2, 3, 4, 6, and 8.)**	**4-5**	**11-14%**
3	NF	A	1	Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b.	0-2	0-6%

3	NF	A	2	Understand a fraction as a number on the number line; represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b and that the endpoint of the part based at 0 locates the number 1/b on the number line. b. Represent the fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.	0-3	0-9%
3	NF	A	3	Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size. a. Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line. b. Recognize and generate simple equivalent fractions, e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent, e.g., by using a visual fraction model. c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram. d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	0-3	0-9%
3	**OA**			**Operations & Algebraic Thinking**	**11-13**	**31-37%**
3	OA	A	0	**Represent and solve problems involving multiplication and division.**	**1-5**	**3-14%**
3	OA	A	1	Interpret products of whole numbers, e.g., interpret 5 x 7 as the total number of objects in 5 groups of 7 objects each. For example, describe a context in which a total number of objects can be expressed as 5 x 7.	0-2	0-6%
3	OA	A	2	Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.	0-2	0-6%

3	OA	A	3	Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.	0-3	0-9%
3	OA	A	4	Determine the unknown whole number in a multiplication or division equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 x ? = 48, 5 = □ div 3, 6 x 6 = ?.	0-3	0-9%
3	OA	B	0	**Understand properties of multiplication and the relationship between multiplication and division.**	**1-4**	**3-11%**
3	OA	B	5	Apply properties of operations as strategies to multiply and divide.2 Examples: If 6 x 4 = 24 is known, then 4 x 6 = 24 is also known. (Commutative property of multiplication.) 3 x 5 x 2 can be found by 3 x 5 = 15, then 15 x 2 = 30, or by 5 x 2 = 10, then 3 × 10 = 30. (Associative property of multiplication.) Knowing that 8 × 5 = 40 and 8 × 2 = 16, one can find 8 × 7 as 8 × (5 + 2) = (8 × 5) + (8 × 2) = 40 + 16 = 56. (Distributive property.)	0-2	0-6%
3	OA	B	6	Understand division as an unknown-factor problem. For example, find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8.	0-2	0-6%
3	OA	C	0	**Multiply and divide within 100.**	**1-3**	**3-9%**
3	OA	C	7	Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 x 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.	1-3	3-9%
3	OA	D	0	**Solve problems involving the four operations, and identify and explain patterns in arithmetic.**	**1-4**	**3-11%**
3	OA	D	8	Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	0-3	0-9%

3	OA	D	9	Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.	0-3	0-9%
3	**G**			**Geometry**	**4-5**	**11-14%**
3	G	A	0	**Reason with shapes and their attributes.**	**4-5**	**11-14%**
3	G	A	1	Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), and that the shared attributes can define a larger category (e.g., quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilateral, draw examples of quadrilaterals that do not below to any of these subcategories.	1-4	3-11%
3	G	A	2	Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 of the area of the shape.	1-4	3-11%
3	**MD**			**Measurement & Data**	**8-10**	**23-29%**
3	MD	A	0	**Solve problems involving measurement and estimation of intervals of time, liquid volumes, and masses of objects.**	**1-4**	**3-11%**
3	MD	A	1	Tell and write time to the nearest minute and measure time intervals in minutes. Solve word problems involving addition and subtraction of time intervals in minutes, e.g., by representing the problem on a number line diagram.	0-3	0-9%
3	MD	A	2	Measure and estimate liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l). Add, subtract, multiply, or divide to solve one-step word problems involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.	0-3	0-9%
3	MD	B	0	**Represent and interpret data.**	**1-4**	**3-11%**

3	MD	B	3	Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one- and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs. For example, draw a bar graph in which each square in the bar graph must represent 5 pets.	0-3	0-9%
3	MD	B	4	Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units - whole numbers, halves, or quarters.	0-3	0-9%
3	MD	C	0	**Geometric measurement: understand concepts of area and relate area to multiplication and to addition.**	**1-3**	**3-9%**
3	MD	C	5	Recognize area as an attribute of plane figures and understand concepts of area measurement. a. A square with side length 1 unit, called “a unit square,” is said to have “one square unit” of area, and can be used to measure area. b. A plane figure which can be covered without gaps or overlaps by n unit squares is said to have an area of n square units.	0-2	0-6%
3	MD	C	6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).	0-2	0-6%
3	MD	C	7	Relate area to the operations of multiplication and addition. a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. b. Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning. c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning. d. Recognize area as additive. Find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.	0-3	0-9%
3	MD	D	0	**Geometric measurement: recognize perimeter as an attribute of plane figures and distinguish between linear and area measures.**	**0-2**	**0-6%**

3	MD	D	8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	0-2	0-6%

Grade 4

Grade	Domain	Cluster	Standard #	Description	Range of operational items	Approx % of total test
4	**NBT**			**Number & Operations in Base Ten**	**7-9**	**19-24%**
4	NBT	A	0	**Generalize place value understanding for multi-digit whole numbers. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)**	**2-6**	**4-16%**
4	NBT	A	1	Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. For example, recognize that 700 ÷ 70 = 10 by applying concepts of place value and division.	0-3	0-8%
4	NBT	A	2	Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	1-3	3-8%
4	NBT	A	3	Use place value understanding to round multi-digit whole numbers to any place.	1-3	3-8%
4	NBT	B	0	**Use place value understanding and properties of operations to perform multi-digit arithmetic. (Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.)**	**3-7**	**8-19%**
4	NBT	B	4	Fluently add and subtract multi-digit whole numbers using the standard algorithm.	1-4	3-9%
4	NBT	B	5	Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	1-4	3-9%

4	NBT	B	6	Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	1-4	3-9%
4	**NF**			**Number & Operations - Fractions**	**8-10**	**22-27%**
4	NF	A	0	**Extend understanding of fraction equivalence and ordering. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)**	**1-5**	**3-14%**
4	NF	A	1	Explain why a fraction a/b is equivalent to a fraction (n ÷ a)/(n ÷ b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	0-2	0-5%
4	NF	A	2	Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.	0-3	0-8%
4	NF	B	0	**Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)**	**1-5**	**3-14%**

4	NF	B	3	Understand a fraction a/b with a > 1 as a sum of fractions 1/b. a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole. b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. Examples: 3/8 = 1/8 + 1/8 + 1/8 ; 3/8 = 1/8 + 2/8 ; 2 1/8 = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8. c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction. d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.	0-4	0-11%
4	NF	B	4	Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of 1/b. For example, use a visual fraction model to represent 5/4 as the product 5 × (1/4), recording the conclusion by the equation 5/4 = 5 × (1/4). b. Understand a multiple of a/b as a multiple of 1/b, and use this understanding to multiply a fraction by a whole number. For example, use a visual fraction model to express 3 × (2/5) as 6 × (1/5), recognizing this product as 6/5. (In general, n × (a/b) = (n × a)/b.) c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. For example, if each person at a party will eat 3/8 of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?	0-3	0-8%
4	NF	C	0	**Understand decimal notation for fractions, and compare decimal fractions. (Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.)**	**1-5**	**3-14%**
4	NF	C	5	Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.4 For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.	0-2	0-5%

4	NF	C	6	Use decimal notation for fractions with denominators 10 or 100. For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram.	0-2	0-5%
4	NF	C	7	Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.	0-2	0-5%
4	**OA**			**Operations and Algebraic Thinking**	**6-8**	**16-22%**
4	OA	A	0	**Use the four operations with whole numbers to solve problems.**	**2-6**	**5-16%**
4	OA	A	1	Interpret a multiplication equation as a comparison, e.g., interpret 35 = 5 x 7 as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.	0-2	0-5%
4	OA	A	2	Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.	0-3	0-8%
4	OA	A	3	Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.	1-4	3-11%
4	OA	B	0	**Gain familiarity with factors and multiples.**	**1-3**	**3-8%**
4	OA	B	4	Find all factor pairs for a whole number in the range 1-100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.	1-3	3-8%
4	OA	C	0	**Generate and analyze patterns.**	**1-3**	**3-8%**

4	OA	C	5	Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.	1-3	3-8%
4	**G**			**Geometry**	**4-6**	**11-16%**
4	G	A	0	**Draw and identify lines and angles, and classify shapes by properties of their lines and angles.**	**4-6**	**11-16%**
4	G	A	1	Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.	1-3	3-8%
4	G	A	2	Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles.	1-3	3-8%
4	G	A	3	Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.	1-2	3-5%
4	**MD**			**Measurement & Data**	**7-9**	**19-24%**
4	MD	A	0	**Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.**	**1-5**	**3-14%**
4	MD	A	1	Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table. For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...	0-2	0-5%

4	MD	A	2	Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.	0-3	0-8%
4	MD	A	3	Apply the area and perimeter formulas for rectangles in real world and mathematical problems. For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.	0-2	0-5%
4	MD	B	0	**Represent and interpret data.**	**0-2**	**0-5%**
4	MD	B	4	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Solve problems involving addition and subtraction of fractions by using information presented in line plots. For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.	0-2	0-5%
4	MD	C	0	**Geometric measurement: understand concepts of angle and measure angles.**	**1-4**	**3-11%**
4	MD	C	5	Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement: a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through 1/360 of a circle is called a “one-degree angle,” and can be used to measure angles. b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.	0-2	0-5%
4	MD	C	6	Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.	0-2	0-5%

4	MD	C	7	Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.	0-2	0-5%

Grade 5

Grade	Domain	Cluster	Standard #	Description	Range of operational items	Approx % of total test
5	**NBT**			**Number & Operations in Base Ten**	**9-11**	**23-28%**
5	NBT	A	0	**Understand the place value system.**	**3-8**	**8-20%**
5	NBT	A	1	Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	0-2	0-5%
5	NBT	A	2	Explain patterns in the number of zeros of the product when multiplying a number by powers of 10, and explain patterns in the placement of the decimal point when a decimal is multiplied or divided by a power of 10. Use whole-number exponents to denote powers of 10.	0-3	0-8%
5	NBT	A	3	Read, write, and compare decimals to thousandths. a. Read and write decimals to thousandths using base-ten numerals, number names, and expanded form, e.g., 347.392 = 3 x 100 + 4 x 10 + 7 x 1 + 3 x (1/10) + 9 x (1/100) + 2 x (1/1000). b. Compare two decimals to thousandths based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.	0-3	0-8%
5	NBT	A	4	Use place value understanding to round decimals to any place.	0-2	0-5%
5	NBT	B	0	**Perform operations with multi-digit whole numbers and with decimals to hundredths.**	**3-8**	**8-20%**
5	NBT	B	5	Fluently multiply multi-digit whole numbers using the standard algorithm.	0-4	0-10%
5	NBT	B	6	Find whole-number quotients of whole numbers with up to four-digit dividends and two-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.	0-4	0-10%

5	NBT	B	7	Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.	0-4	0-10%
5	**NF**			**Number & Operations - Fractions**	**9-11**	**23-28%**
5	NF	A	0	**Use equivalent fractions as a strategy to add and subtract fractions.**	**2-7**	**5-18%**
5	NF	A	1	Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference of fractions with like denominators. For example, 2/3 + 5/4 = 8/12 + 15/12 = 23/12. (In general, a/b + c/d = (ad + bc)/bd.)	1-4	3-10%
5	NF	A	2	Solve word problems involving addition and subtraction of fractions referring to the same whole, including cases of unlike denominators, e.g., by using visual fraction models or equations to represent the problem. Use benchmark fractions and number sense of fractions to estimate mentally and assess the reasonableness of answers. For example, recognize an incorrect result 2/5 + 1/2 = 3/7, by observing that 3/7 < 1/2.	1-4	3-10%
5	NF	B	0	**Apply and extend previous understandings of multiplication and division to multiply and divide fractions.**	**4-9**	**10-23%**
5	NF	B	3	Interpret a fraction as division of the numerator by the denominator (a/b = a ÷ b). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret 3/4 as the result of dividing 3 by 4, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?	0-2	0-5%

5	NF	B	4	Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction. a. Interpret the product (a/b) × q as a parts of a partition of q into b equal parts; equivalently, as the result of a sequence of operations a × q ÷ b. For example, use a visual fraction model to show (2/3) × 4 = 8/3, and create a story context for this equation. Do the same with (2/3) × (4/5) = 8/15. (In general, (a/b) × (c/d) = ac/bd.) b. Find the area of a rectangle with fractional side lengths by tiling it with unit squares of the appropriate unit fraction side lengths, and show that the area is the same as would be found by multiplying the side lengths. Multiply fractional side lengths to find areas of rectangles, and represent fraction products as rectangular areas.	0-3	0-8%
5	NF	B	5	Interpret multiplication as scaling (resizing), by: a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication. b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence a/b = (n×a)/(n×b) to the effect of multiplying a/b by 1.	0-2	0-5%
5	NF	B	6	Solve real world problems involving multiplication of fractions and mixed numbers, e.g., by using visual fraction models or equations to represent the problem.	0-2	0-5%

5	NF	B	7	Apply and extend previous understandings of division to divide unit fractions by whole numbers and whole numbers by unit fractions. a. Interpret division of a unit fraction by a non-zero whole number, and compute such quotients. For example, create a story context for (1/3) ÷ 4, and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that (1/3) ÷ 4 = 1/12 because (1/12) × 4 = 1/3. b. Interpret division of a whole number by a unit fraction, and compute such quotients. For example, create a story context for 4 ÷ (1/5), and use a visual fraction model to show the quotient. Use the relationship between multiplication and division to explain that 4 ÷ (1/5) = 20 because 20 × (1/5) = 4. c. Solve real world problems involving division of unit fractions by non-zero whole numbers and division of whole numbers by unit fractions, e.g., by using visual fraction models and equations to represent the problem. For example, how much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 1/3-cup servings are in 2 cups of raisins?	0-2	0-5%
5	**OA**			**Operations & Algebraic Thinking**	**5-7**	**13-18%**
5	OA	A	0	**Write and interpret numerical expressions.**	**3-6**	**8-15%**
5	OA	A	1	Use parentheses, brackets, or braces in numerical expressions, and evaluate expressions with these symbols.	1-4	3-10%
5	OA	A	2	Write simple expressions that record calculations with numbers, and interpret numerical expressions without evaluating them. For example, express the calculation "add 8 and 7, then multiply by 2" as 2 x (8 + 7). Recognize that 3 x (18932 + 921) is three times as large as 18932 + 921, without having to calculate the indicated sum or product.	1-4	3-10%
5	OA	B	0	**Analyze patterns and relationships.**	**1-4**	**3-10%**
5	OA	B	3	Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.	1-4	3-10%

5	**G**			**Geometry**	**5-7**	**13-18%**
5	G	A	0	**Graph points on the coordinate plane to solve real-world and mathematical problems.**	**1-6**	**3-15%**
5	G	A	1	Use a pair of perpendicular number lines, called axes, to define a coordinate system, with the intersection of the lines (the origin) arranged to coincide with the 0 on each line and a given point in the plane located by using an ordered pair of numbers, called its coordinates. Understand that the first number indicates how far to travel from the origin in the direction of one axis, and the second number indicates how far to travel in the direction of the second axis, with the convention that the names of the two axes and the coordinates correspond (e.g., x-axis and x-coordinate, y-axis and y-coordinate).	0-3	0-8%
5	G	A	2	Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.	0-3	0-8%
5	G	B	0	**Classify two-dimensional figures into categories based on their properties.**	**1-6**	**3-15%**
5	G	B	3	Understand that attributes belonging to a category of two-dimensional figures also belong to all subcategories of that category. For example, all rectangles have four right angles and squares are rectangles, so all squares have four right angles.	0-3	0-8%
5	G	B	4	Classify two-dimensional figures in a hierarchy based on properties.	0-3	0-8%
5	**MD**			**Measurement & Data**	**7-9**	**18-23%**
5	MD	A	0	**Convert like measurement units within a given measurement system.**	**1-4**	**3-10%**
5	MD	A	1	Convert among different-sized standard measurement units within a given measurement system (e.g., convert 5 cm to 0.05 m), and use these conversions in solving multi-step, real world problems.	1-4	3-10%
5	MD	B	0	**Represent and interpret data.**	**0-3**	**0-8%**
5	MD	B	2	Make a line plot to display a data set of measurements in fractions of a unit (1/2, 1/4, 1/8). Use operations on fractions for this grade to solve problems involving information presented in line plots. For example, given different measurements of liquid each beaker would contain if the total amount in all the beakers were redistributed equally.	0-3	0-8%

5	MD	C	0	**Geometric measurement: understand concepts of volume and relate volume to multiplication and to addition.**	**1-4**	**3-10%**
5	MD	C	3	Recognize volume as an attribute of solid figures and understand concepts of volume measurement. a. A cube with side length 1 unit, called a "unit cube," is said to have "one cubic unit" of volume, and can be used to measure volume. b. A solid figure which can be packed without gaps or overlaps using n unit cubes is said to have a volume of n cubic units.	0-2	0-5%
5	MD	C	4	Measure volumes by counting unit cubes, using cubic cm, cubic in, cubic ft, and improvised units.	0-2	0-5%
5	MD	C	5	Relate volume to the operations of multiplication and addition and solve real world and mathematical problems involving volume. a. Find the volume of a right rectangular prism with whole-number side lengths by packing it with unit cubes, and show that the volume is the same as would be found by multiplying the edge lengths, equivalently by multiplying the height by the area of the base. Represent threefold whole-number products as volumes, e.g., to represent the associative property of multiplication. b. Apply the formulas $V = l \times w \times h$ and $V = b \times h$ for rectangular prisms to find volumes of right rectangular prisms with whole-number edge lengths in the context of solving real world and mathematical problems. c. Recognize volume as additive. Find volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to solve real world problems.	1-4	3-10%

Grade 6

Grade	Domain	Cluster	Standard #	Description	Range of operational items	Approx % of total test
6	**NS**			**Number & Operations in Base Ten**	**9-11**	**21-27%**
6	NS	A	0	**Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**	**1-3**	**2-7%**
6	NS	A	1	Interpret and compute quotients of fractions, and solve word problems involving division of fractions by fractions, e.g., by using visual fraction models and equations to represent the problem. For example, create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?	1-3	2-7%
6	NS	B	0	**Compute fluently with multi-digit numbers and find common factors and multiples.**	**2-8**	**5-19%**
6	NS	B	2	Fluently divide multi-digit numbers using the standard algorithm.	0-4	0-10%
6	NS	B	3	Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.	0-4	0-10%
6	NS	B	4	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor. For example, express 36 + 8 as 4 (9 + 2).	0-2	0-5%
6	NS	C	0	**Apply and extend previous understandings of numbers to the system of rational numbers.**	**2-8**	**5-19%**

6	NS	C	5	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.	0-2	0-5%
6	NS	C	6	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.	0-2	0-5%
6	NS	C	7	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. For example, interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right. b. Write, interpret, and explain statements of order for rational numbers in real-world contexts. For example, write –3°C > –7°C to express the fact that –3°C is warmer than –7°C. c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. For example, for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars. d. Distinguish comparisons of absolute value from statements about order. For example, recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.	0-2	0-5%
6	NS	C	8	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.	0-3	0-7%

6	**RP**			**Ratios & Proportional Relationships**	**6-8**	**14-19%**
6	RP	A	0	**Understand ratio concepts and use ratio reasoning to solve problems.**	**6-8**	**14-19%**
6	RP	A	1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." For every vote candidate A received, candidate C received nearly three votes."	0-3	0-7%
6	RP	A	2	Understand the concept of a unit rate a/b associated with a ratio a:b with b not equal 0, and use rate language in the context of a ratio relationship. For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."	0-3	0-7%
6	RP	A	3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed? c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.	2-7	5-17%
6	**EE**			**Expressions & Equations**	**12-14**	**29-33%**
6	EE	A	0	**Apply and extend previous understandings of arithmetic to algebraic expressions.**	**3-8**	**7-19%**
6	EE	A	1	Write and evaluate numerical expressions involving whole-number exponents.	0-3	0-7%

6	EE	A	2	Write, read, and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. For example, express the calculation "Subtract y from 5" as 5 - y. b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. For example, describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms. c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas V = s3 and A = 6 s2 to find the volume and surface area of a cube with sides of length s = 1/2.	0-5	0-12%
6	EE	A	3	Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y.	0-3	0-7%
6	EE	A	4	Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them).For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.	0-3	0-7%
6	EE	B	0	**Reason about and solve one-variable equations and inequalities.**	**3-8**	**7-19%**
6	EE	B	5	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.	0-3	0-7%
6	EE	B	6	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number, or, depending on the purpose at hand, any number in a specified set.	0-3	0-7%
6	EE	B	7	Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.	0-5	0-12%

6	EE	B	8	Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.	0-3	0-7%
6	EE	C	0	**Represent and analyze quantitative relationships between dependent and independent variables.**	**0-3**	**0-7%**
6	EE	C	9	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.	0-3	0-7%
6	**G**			**Geometry**	**5-7**	**12-17%**
6	G	A	0	**Solve real-world and mathematical problems involving area, surface area, and volume.**	**5-7**	**12-17%**
6	G	A	1	Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.	0-3	0-7%
6	G	A	2	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.	0-2	0-5%
6	G	A	3	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.	0-2	0-5%
6	G	A	4	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.	0-2	0-5%

6	**SP**			**Statistics & Probability**	**5-7**	**12-17%**
6	SP	A	0	**Develop understanding of statistical variability.**	**0-5**	**0-12%**
6	SP	A	1	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. For example, "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.	0-2	0-5%
6	SP	A	2	Understand that a set of data collected to answer a statistical question has a distribution which can be described by its center, spread, and overall shape.	0-2	0-5%
6	SP	A	3	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.	0-2	0-5%
6	SP	B	0	**Summarize and describe distributions.**	**2-7**	**5-17%**
6	SP	B	4	Display numerical data in plots on a number line, including dot plots, histograms, and box plots.	0-3	0-7%
6	SP	B	5	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.	1-5	2-12%

Grade 7

Grade	Domain	Cluster	Standard #	Description	Range of operational items	Approx % of total test
7	**NS**			**The Number System**	**9-11**	**20-24%**
7	NS	A	0	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**	**9-11**	**20-24%**
7	NS	A	1	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged. b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.	1-4	2-8%
7	NS	A	2	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real-world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.	1-4	2-8%

7	NS	A	3	Solve real-world and mathematical problems involving the four operations with rational numbers.	2-8	4-18%
7	**RP**			**Ratios & Proportions**	**8-10**	**18-22%**
7	RP	A	0	**Analyze proportional relationships and use them to solve real-world and mathematical problems.**	**8-10**	**18-22%**
7	RP	A	1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. For example, if a person walks 1/2 mile in each 1/4 hour, compute the unit rate as the complex fraction {1/2}/{1/4} miles per hour, equivalently 2 miles per hour.	1-4	2-8%
7	RP	A	2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. For example, if total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn. d. Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate.	1-5	2-11%
7	RP	A	3	Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.	2-7	4-16%
7	**EE**			**Expressions & Equations**	**10-12**	**22-27%**
7	EE	A	0	**Use properties of operations to generate equivalent expressions.**	**1-7**	**2-16%**
7	EE	A	1	Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	0-4	0-9%
7	EE	A	2	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. For example, a + 0.05a = 1.05a means that "increase by 5%" is the same as "multiply by 1.05."	0-4	0-9%
7	EE	B	0	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**	**5-11**	**11-24%**

7	EE	B	3	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.	2-9	4-20%
7	EE	B	4	Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form px + q = r and p(x + q) = r, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? b. Solve word problems leading to inequalities of the form px + q > r or px + q < r, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. For example: As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.	2-9	4-20%
7	**G**			**Geometry**	**8-10**	**18-23%**
7	G	A	0	**Draw, construct, and describe geometrical figures and describe the relationships between them.**	**1-7**	**2-16%**
7	G	A	1	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.	0-2	0-4%
7	G	A	2	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.	0-3	0-7%
7	G	A	3	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.	0-3	0-7%

7	G	B	0	**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**	**3-9**	**7-20%**
7	G	B	4	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.	1-3	2-7%
7	G	B	5	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.	1-4	2-9%
7	G	B	6	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.	1-4	2-9%
7	**SP**			**Statistics & Probability**	**5-7**	**12-17%**
7	SP	A	0	**Use random sampling to draw inferences about a population.**	**0-2**	**0-4%**
7	SP	A	1	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.	0-2	0-4%
7	SP	A	2	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. For example, estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.	0-2	0-4%
7	SP	B	0	**Draw informal comparative inferences about two populations.**	**0-2**	**0-5%**
7	SP	B	3	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. For example, the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.	0-2	0-4%
7	SP	B	4	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. For example, decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.	0-2	0-4%

7	SP	C	0	**Investigate chance processes and develop, use, and evaluate probability models.**	**1-5**	**3-11%**
7	SP	C	5	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.	0-2	0-4%
7	SP	C	6	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. For example, when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.	0-2	0-4%
7	SP	C	7	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected. b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. For example, find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?	1-3	2-7%
7	SP	C	8	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. For example, use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?	0-3	0-7%

Grade 8

Grade	Domain	Cluster	Standard #	Description	Range of operational items	Approx % of total test
8	**NS**			**The Number System**	**4-6**	**9-13%**
8	NS	A	0	**Know that there are numbers that are not rational, and approximate them by rational numbers.**	**4-6**	**9-13%**
8	NS	A	1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.	1-5	2-11%
8	NS	A	2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., pi^2). For example, by truncating the decimal expansion of √2, show that √2 is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.	1-5	2-11%
8	**F**			**Functions**	**9-11**	**20-24%**
8	F	A	0	**Define, evaluate, and compare functions.**	**3-8**	**6-17%**
8	F	A	1	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.	0-3	0-6%
8	F	A	2	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.	0-3	0-6%
8	F	A	3	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph	0-3	0-6%

				contains the points (1,1), (2,4) and (3,9), which are not on a straight line.		
8	F	B	0	**Use functions to model relationships between quantities.**	**3-8**	**6-17%**
8	F	B	4	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.	1-5	2-11%
8	F	B	5	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.	0-3	0-6%
8	**EE**			**Expressions & Equations**	**13-15**	**28-32%**
8	EE	A	0	**Work with radicals and integer exponents.**	**3-10**	**6-21%**
8	EE	A	1	Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 3^2 x 3^-5 = 3^-3 = 1/3^3 = 1/27.	1-4	2-9%
8	EE	A	2	Use square root and cube root symbols to represent solutions to equations of the form x^2 = p and x^3 = p, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.	1-4	2-9%
8	EE	A	3	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. For example, estimate the population of the United States as 3 x 10^8 and the population of the world as 7 × 10^9, and determine that the world population is more than 20 times larger.	0-2	0-4%

8	EE	A	4	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.	1-4	2-9%
8	EE	B	0	**Understand the connections between proportional relationships, lines, and linear equations.**	**0-4**	**0-9%**
8	EE	B	5	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.	0-3	0-6%
8	EE	B	6	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.	0-3	0-6%
8	EE	C	0	**Analyze and solve linear equations and pairs of simultaneous linear equations.**	**3-10**	**6-21%**
8	EE	C	7	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form x = a, a = a, or a = b results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.	2-8	4-17%

8	EE	C	8	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.	1-3	2-6%
8	**G**			**Geometry**	**9-11**	**19-23%**
8	G	A	0	**Understand congruence and similarity using physical models, transparencies, or geometry software.**	**4-9**	**8-19%**
8	G	A	1	Verify experimentally the properties of rotations, reflections, and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.	0-2	0-4%
8	G	A	2	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.	0-3	0-6%
8	G	A	3	Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.	0-3	0-6%
8	G	A	4	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.	0-3	0-6%
8	G	A	5	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.	0-5	0-11%

8	G	B	0	**Understand and apply the Pythagorean Theorem.**	**1-4**	**4-11%**
8	G	B	6	Explain a proof of the Pythagorean Theorem and its converse.	0-1	0-1%
8	G	B	7	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.	1-2	2-4%
8	G	B	8	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.	0-2	0-4%
8	G	C	0	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**	**1-4**	**4-11%**
8	G	C	9	Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.	1-4	2-9%
8	**SP**			**Statistics & Probability**	**7-9**	**15-19%**
8	SP	A	0	**Investigate patterns of association in bivariate data.**	**7-9**	**15-19%**
8	SP	A	1	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.	0-3	0-6%
8	SP	A	2	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.	0-3	0-6%
8	SP	A	3	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.	0-3	0-6%

8	SP	A	4	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. For example, collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?	0-3	0-6%

Grade 9

Grade	Domain	Cluster	Standard #	Description	Range of operational items	Approx % of total test
9	**N**			**Number and Quantity**	**6-8**	**17-23%**
9	**NRN**			**The Real Number System**	**2-5**	**6-14%**
9	NRN	A	0	**Extend the properties of exponents to rational exponents.**	**1-4**	**3-11%**
9	NRN	A	1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 5^{1/3} to be the cube root of 5 because we want (5^{1/3})^3 = 5^{(1/3)3} to hold, so (5^{1/3})^3 must equal 5.	0-3	0-9%
9	NRN	A	2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	0-4	0-11%
9	NRN	B	0	**Use properties of rational and irrational numbers.**	**0-3**	
9	NRN	B	3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	0-3	0-9%
9	**NQ**			**Quantities**	**2-5**	**6-14%**
9	NQ	A	0	**Reason quantitatively and use units to solve problems.**	**2-5**	**6-14%**
9	NQ	A	1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	1-3	3-9%
9	NQ	A	2	Define appropriate quantities for the purpose of descriptive modeling.	0-3	0-9%
9	NQ	A	3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	0-3	0-9%
9	NQ	B	0	**Understand and apply the mathematics of voting.**	**0-1**	**0-3%**

9	NQ	B	IA3	Understand, analyze, apply, and evaluate some common voting and analysis methods in addition to majority and plurality, such as runoff, approval, the so-called instant-runoff voting (IRV) method, the Borda method and the Condorcet method.	0-1	0-3%
9	NQ	C	0	**Understand and apply some basic mathematics of information processing and the Internet.**	**0-1**	**0-3%**
9	NQ	C	IA5	Understand and apply elementary set theory and logic as used in simple Internet searches.	0-1	0-3%
9	**F**			**Functions**	**6-8**	**17-23%**
9	**FIF**			**Interpreting Functions**	**1-5**	**3-14%**
9	FIF	A	0	**Understand the concept of a function and use function notation.**	**0-3**	**0-9%**
9	FIF	A	1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	0-2	0-6%
9	FIF	A	2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	0-2	0-6%
9	FIF	A	3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1) for n ≥ 1.	0-1	0-3%
9	FIF	B	0	**Interpret functions that arise in applications in terms of the context.**	**0-4**	**0-11%**
9	FIF	B	4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	0-2	0-6%
9	FIF	B	5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.	0-1	0-3%
9	FIF	B	6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	0-2	0-6%

9	FIF	C	0	**Analyze functions using different representations.**	**0-3**	**0-9%**
9	FIF	C	7abce	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	0-3	0-9%
9	FIF	C	8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)^t, y = (0.97)^t, y = (1.01)^{12t}, y = (1.2)^{t/10}, and classify them as representing exponential growth or decay.	0-2	0-6%
9	FIF	C	9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	0-2	0-6%
9	**FBF**			**Building Functions**	**1-4**	**3-11%**
9	FBF	A	0	**Build a function that models a relationship between two quantities.**	**0-4**	**0-11%**
9	FBF	A	1ab	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.	0-2	0-6%
9	FBF	A	2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	0-2	0-6%
9	FBF	B	0	**Build new functions from existing functions.**	**0-2**	**0-6%**

9	FBF	B	3	Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	0-2	0-6%
9	FBF	B	4a	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = 2x^3 or f(x) = (x + 1)/(x - 1) for x ≠ 1.	0-1	0-3%
9	**FLE**			**Linear, Quadratic, & Exponential Models**	**1-3**	**3-9%**
9	FLE	A	0	**Construct and compare linear, quadratic, and exponential models and solve problems.**	**0-3**	**0-9%**
9	FLE	A	1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	0-2	0-6%
9	FLE	A	2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	0-2	0-6%
9	FLE	A	3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	0-2	0-6%
9	FLE	B	0	**Interpret expressions for functions in terms of the situation they model.**	**0-1**	**0-3%**
9	FLE	B	5	Interpret the parameters in a linear or exponential function in terms of a context.	0-1	0-3%
9	**A**			**Algebra**	**10-12**	**29-34%**
9	**ASSE**			**Seeing Structure in Expressions**	**1-3**	**3-9%**
9	ASSE	A	0	**Interpret the structure of expressions.**	**0-2**	**0-6%**
9	ASSE	A	1	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)^n as the product of P and a factor not depending on P.	0-2	0-6%

9	ASSE	A	2	Use the structure of an expression to identify ways to rewrite it. For example, see x^4 - y^4 as (x^2)^2 - (y^2)^2, thus recognizing it as a difference of squares that can be factored as (x^2 - y^2)(x^2 + y^2).	0-2	0-6%
9	ASSE	B	0	**Write expressions in equivalent forms to solve problems.**	**0-2**	**0-6%**
9	ASSE	B	3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.15^{1/12})^{12t} ≈ 1.012^{12t} to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.	0-2	0-6%
9	ASSE	B	4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.	0-1	0-3%
9	**AAPR**			**Arithmetic with Polynomials & Rational Expressions**	**1-4**	**3-11%**
9	AAPR	A	0	**Perform arithmetic operations on polynomials.**	**0-3**	**0-9%**
9	AAPR	A	1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	0-3	0-9%
9	AAPR	D	0	**Rewrite rational expressions.**	**0-1**	**0-3%**
9	AAPR	D	6	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.	0-1	0-3%
9	**ACED**			**Creating Equations**	**1-5**	**3-14%**
9	ACED	A	0	**Create equations that describe numbers or relationships.**	**1-5**	**3-14%**
9	ACED	A	1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	0-3	0-9%
9	ACED	A	2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	0-3	0-9%

9	ACED	A	3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	0-2	0-6%
9	ACED	A	4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	0-2	0-6%
9	**AREI**			**Reasoning with Equations & Inequalities**	**1-5**	**3-14%**
9	AREI	A	0	**Understand solving equations as a process of reasoning and explain the reasoning.**	**0-2**	**0-6%**
9	AREI	A	1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	0-2	0-6%
9	AREI	B	0	**Solve equations and inequalities in one variable.**	**1-4**	**3-11%**
9	AREI	B	3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	0-3	0-9%
9	AREI	B	4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x - p)^2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	0-2	0-6%
9	AREI	C	0	**Solve systems of equations**	**0-2**	**0-6%**
9	AREI	C	5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	0-1	0-3%
9	AREI	C	6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	0-1	0-3%
9	AREI	C	7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle x^2 + y^2 = 3.	0-1	0-3%
9	AREI	D	0	**Represent and solve equations and inequalities graphically.**	**0-2**	**0-6%**

9	AREI	D	10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	0-1	0-3%
9	AREI	D	11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	0-1	0-3%
9	AREI	D	12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	0-1	0-3%
9	**G**			**Geometry**	**4-6**	**11-17%**
9	**GCO**			**Congruence**	**1-4**	**3-11%**
9	GCO	A	0	**Experiment with transformations in the plane.**	**0-3**	**0-9%**
9	GCO	A	1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	0-2	0-6%
9	GCO	A	2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	0-2	0-6%
9	GCO	A	3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	0-2	0-6%
9	GCO	A	4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	0-2	0-6%
9	GCO	A	5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	0-2	0-6%
9	GCO	B	0	**Understand congruence in terms of rigid motions.**	**0-2**	**0-6%**
9	GCO	B	6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	0-2	0-6%

9	GCO	B	7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	0-2	0-6%
9	GCO	C	0	**Prove geometric theorems.**	**0-3**	**0-9%**
9	GCO	C	9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	0-2	0-6%
9	GCO	C	10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	0-2	0-6%
9	GCO	C	11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	0-2	0-6%
9	**GSRT**			**Similarity, Right Triangles, & Trigonometry**	**0-4**	**0-11%**
9	GSRT	A	0	**Understand similarity in terms of similarity transformations.**	**0-2**	**0-6%**
9	GSRT	A	1	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	0-2	0-6%
9	GSRT	A	2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	0-2	0-6%
9	GSRT	B	0	**Prove theorems involving similarity.**	**0-2**	**0-6%**
9	GSRT	B	4	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	0-2	0-6%
9	GSRT	B	5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	0-2	0-6%
9	GSRT	C	0	**Define trigonometric ratios and solve problems involving right triangles.**	**0-3**	**0-9%**

9	GSRT	C	6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	0-2	0-6%
10	GSRT	C	7	Explain and use the relationship between the sine and cosine of complementary angles.	0	0%
9	GSRT	C	8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	0-2	0-6%
9	**GC**			**Circles**	**0-2**	**0-6%**
9	GC	A	0	**Understand and apply theorems about circles.**	**0-2**	**0-6%**
9	GC	A	1	Prove that all circles are similar.	0-1	0-3%
9	GC	A	2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	0-1	0-3%
9	GC	A	3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	0-1	0-3%
9	GC	B	0	**Find arc lengths and areas of sectors of circles.**	**0-1**	**0-3%**
9	GC	B	5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	0-1	0-3%
9	**GGPE**			**Expressing Geometric Properties with Equations**	**0-3**	**0-9%**
9	GGPE	B	0	**Use coordinates to prove simple geometric theorems algebraically.**	**0-3**	**0-9%**
9	GGPE	B	4	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,sqt3) lies on the circle centered at the origin and containing the point (0, 2).	0-2	0-6%
9	GGPE	B	5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	0-2	0-6%
9	GGPE	B	6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	0-2	0-6%
9	GGPE	B	7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	0-2	0-6%

9	**GGMD**			**Geometric Measurement & Dimension**	**0-2**	**0-6%**
9	GGMD	A	0	**Explain volume formulas and use them to solve problems.**	**0-2**	**0-6%**
9	GGMD	A	1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	0-2	0-6%
9	GGMD	A	3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	0-2	0-6%
9	GGMD	B	0	**Visualize relationships between two-dimensional and three-dimensional objects.**	**0-2**	**0-6%**
9	GGMD	B	4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	0-1	0-3%
9	GGMD	B	IA7	Plot points in three-dimensions.	0-1	0-3%
9	**GMG**			**Modeling with Geometry**	**0-2**	**0-6%**
9	GMG	A	0	**Apply geometric concepts in modeling situations.**	**0-2**	**0-6%**
9	GMG	A	1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	0-2	0-6%
9	GMG	A	2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	0-1	0-3%
9	GMG	A	3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	0-1	0-3%
9	GMG	B	0	**Use diagrams consisting of vertices and edges (vertex-edge graphs) to model and solve problems related to networks.**	**0-1**	**0-3%**
9	GMG	B	IA8	Understand, analyze, evaluate, and apply vertex-edge graphs to model and solve problems related to paths, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings.	0-1	0-3%
9	GMG	B	IA9	Model and solve problems using at least two of the following fundamental graph topics and models: Euler paths and circuits, Hamilton paths and circuits, the traveling salesman problem (TSP), minimum spanning trees, critical paths, vertex coloring.	0-1	0-3%
9	GMG	B	IA10	Compare and contrast vertex-edge graph topics and models in terms of: properties algorithms optimization types of problems that can be solved	0-1	0-3%

9	**S**			**Statistics & Probability**	**4-6**	**11-17%**
9	**SID**			**Interpreting Categorical & Quantitative Data**	**1-5**	**3-14%**
9	SID	A	0	**Summarize, represent, and interpret data on a single count or measurement variable.**	**0-3**	**0-9%**
9	SID	A	1	Represent data with plots on the real number line (dot plots, histograms, and box plots).	0-3	0-9%
9	SID	A	2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	0-2	0-6%
9	SID	A	3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	0-2	0-6%
9	SID	B	0	**Summarize, represent, and interpret data on two categorical and quantitative variables.**	**0-3**	**0-9%**
9	SID	B	5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	0-2	0-6%
9	SID	B	6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	0-2	0-6%
9	SID	C	0	**Interpret linear models.**	**0-3**	**0-9%**
9	SID	C	7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	0-2	0-6%
9	SID	C	8	Compute (using technology) and interpret the correlation coefficient of a linear fit.	0-1	0-3%
9	SID	C	9	Distinguish between correlation and causation.	0-1	0-3%
9	**SCP**			**Conditional Probability & the Rules of Probability**	**0-4**	**0-11%**
9	SCP	A	0	**Understand independence and conditional probability and use them to interpret data.**	**0-4**	**0-11%**
9	SCP	A	1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	0-3	0-9%

9	SCP	A	2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	0-2	0-6%
9	SCP	A	3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	0-2	0-6%
9	SCP	A	4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.	0-2	0-6%
9	SCP	A	5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.	0-2	0-6%
9	SCP	B	0	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**	**0-2**	**0-6%**
9	SCP	B	6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	0-1	0-3%
9	SCP	B	7	Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.	0-1	0-3%

Grade 10

Grade	Domain	Cluster	Standard #	Description	Range of operational items	Approx % of total test
10	**N**			**Number and Quantity**	**6-8**	**17-23%**
10	**NRN**			**The Real Number System**	**2-5**	**6-14%**
10	NRN	A	0	**Extend the properties of exponents to rational exponents.**	**1-4**	**3-11%**
10	NRN	A	1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 5^{1/3} to be the cube root of 5 because we want (5^{1/3})^3 = 5^{(1/3)3} to hold, so (5^{1/3})^3 must equal 5.	0-3	0-9%
10	NRN	A	2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	0-4	0-11%
10	NRN	B	0	**Use properties of rational and irrational numbers.**	**0-3**	**0-9%**
10	NRN	B	3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	0-3	0-9%
10	**NQ**			**Quantities**	**2-5**	**6-14%**
10	NQ	A	0	**Reason quantitatively and use units to solve problems.**	**2-5**	**6-14%**
10	NQ	A	1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	1-3	3-9%
10	NQ	A	2	Define appropriate quantities for the purpose of descriptive modeling.	0-3	0-9%
10	NQ	A	3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	0-3	0-9%
10	NQ	B	0	**Understand and apply the mathematics of voting.**	**0-1**	**0-3%**

10	NQ	B	IA3	Understand, analyze, apply, and evaluate some common voting and analysis methods in addition to majority and plurality, such as runoff, approval, the so-called instant-runoff voting (IRV) method, the Borda method and the Condorcet method.	0-1	0-3%
10	NQ	C	0	**Understand and apply some basic mathematics of information processing and the Internet.**	**0-1**	**0-3%**
10	NQ	C	IA5	Understand and apply elementary set theory and logic as used in simple Internet searches.	0-1	0-3%
10	**F**			**Functions**	**5-7**	**14-20%**
10	**FIF**			**Interpreting Functions**	**1-3**	**3-9%**
10	FIF	A	0	**Understand the concept of a function and use function notation.**	**0-3**	**0-9%**
10	FIF	A	1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	0-2	0-6%
10	FIF	A	2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	0-2	0-6%
10	FIF	A	3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1) for n ≥ 1.	0-1	0-3%
10	FIF	B	0	**Interpret functions that arise in applications in terms of the context.**	**0-3**	**0-9%**
10	FIF	B	4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	0-2	0-6%

10	FIF	B	5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.	0-1	0-3%
10	FIF	B	6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	0-2	0-6%
10	FIF	C	0	**Analyze functions using different representations.**	**0-3**	**0-9%**
10	FIF	C	7abce	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	0-3	0-9%
10	FIF	C	8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)^t, y = (0.97)^t, y = (1.01)^{12t}, y = (1.2)^{t/10}, and classify them as representing exponential growth or decay.	0-2	0-6%
10	FIF	C	9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	0-2	0-6%
10	**FBF**			**Building Functions**	**1-3**	**3-9%**
10	FBF	A	0	**Build a function that models a relationship between two quantities.**	**0-3**	**0-9%**

10	FBF	A	1ab	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.	0-2	0-6%
10	FBF	A	2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	0-2	0-6%
10	FBF	B	0	**Build new functions from existing functions.**	**0-2**	**0-6%**
10	FBF	B	3	Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	0-2	0-6%
10	FBF	B	4a	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = 2x^3 or f(x) = (x + 1)/(x - 1) for x ≠ 1.	0-1	0-3%
10	**FLE**			**Linear, Quadratic, & Exponential Models**	**1-3**	**3-9%**
10	FLE	A	0	**Construct and compare linear, quadratic, and exponential models and solve problems.**	**0-3**	**0-9%**
10	FLE	A	1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	0-2	0-6%
10	FLE	A	2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	0-2	0-6%

10	FLE	A	3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	0-2	0-6%
10	FLE	B	0	**Interpret expressions for functions in terms of the situation they model.**	**0-1**	**0-3%**
10	FLE	B	5	Interpret the parameters in a linear or exponential function in terms of a context.	0-1	0-3%
10	**A**			**Algebra**	**6-8**	**17-23%**
10	**ASSE**			**Seeing Structure in Expressions**	**1-3**	**3-9%**
10	ASSE	A	0	**Interpret the structure of expressions.**	**0-2**	**0-6%**
10	ASSE	A	1	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)^n as the product of P and a factor not depending on P.	0-2	0-6%
10	ASSE	A	2	Use the structure of an expression to identify ways to rewrite it. For example, see x^4 - y^4 as (x^2)^2 - (y^2)^2, thus recognizing it as a difference of squares that can be factored as (x^2 - y^2)(x^2 + y^2).	0-2	0-6%
10	ASSE	B	0	**Write expressions in equivalent forms to solve problems.**	**0-2**	**0-6%**
10	ASSE	B	3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.15^{1/12})^{12t} ≈ 1.012^{12t} to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.	0-2	0-6%
10	ASSE	B	4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.	0-1	0-3%
10	**AAPR**			**Arithmetic with Polynomials & Rational Expressions**	**1-4**	**3-11%**
10	AAPR	A	0	**Perform arithmetic operations on polynomials.**	**0-3**	**0-9%**

10	AAPR	A	1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	0-3	0-9%
10	AAPR	D	0	**Rewrite rational expressions.**	**0-1**	**0-3%**
10	AAPR	D	6	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.	0-1	0-3%
10	**ACED**			**Creating Equations**	**1-4**	**3-11%**
10	ACED	A	0	**Create equations that describe numbers or relationships.**	**1-4**	**3-11%**
10	ACED	A	1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	0-2	0-6%
10	ACED	A	2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	0-2	0-6%
10	ACED	A	3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	0-2	0-6%
10	ACED	A	4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	0-2	0-6%
10	**AREI**			**Reasoning with Equations & Inequalities**	**1-4**	**3-11%**
10	AREI	A	0	**Understand solving equations as a process of reasoning and explain the reasoning.**	**0-2**	**0-6%**
10	AREI	A	1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	0-2	0-6%
10	AREI	A	2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	0	0%

10	AREI	B	0	**Solve equations and inequalities in one variable.**	**1-3**	**3-9%**
10	AREI	B	3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	0-2	0-6%
10	AREI	B	4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x - p)^2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	0-2	0-6%
10	AREI	C	0	**Solve systems of equations**	**0-2**	**0-6%**
10	AREI	C	5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	0-1	0-3%
10	AREI	C	6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	0-1	0-3%
10	AREI	C	7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle x^2 + y^2 = 3.	0-1	0-3%
10	AREI	D	0	**Represent and solve equations and inequalities graphically.**	**0-2**	**0-6%**
10	AREI	D	10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	0-1	0-3%
10	AREI	D	11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	0-1	0-3%

10	AREI	D	12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	0-1	0-3%
10	**G**			**Geometry**	**9-11**	**26-32%**
10	**GCO**			**Congruence**	**1-6**	**3-17%**
10	GCO	A	0	**Experiment with transformations in the plane.**	**0-3**	**0-9%**
10	GCO	A	1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	0-2	0-6%
10	GCO	A	2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	0-2	0-6%
10	GCO	A	3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	0-2	0-6%
10	GCO	A	4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	0-2	0-6%
10	GCO	A	5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	0-2	0-6%
10	GCO	B	0	**Understand congruence in terms of rigid motions.**	**0-2**	**0-6%**
10	GCO	B	6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	0-2	0-6%
10	GCO	B	7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	0-2	0-6%
10	GCO	B	8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	0-2	0-6%

10	GCO	C	0	**Prove geometric theorems.**	**0-3**	**0-9%**
10	GCO	C	9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	0-2	0-6%
10	GCO	C	10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	0-2	0-6%
10	GCO	C	11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	0-2	0-6%
10	GCO	D	0	**Make geometric constructions.**	**0-2**	**0-6%**
10	GCO	D	12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	0-2	0-6%
10	GCO	D	13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	0-1	0-3%
10	**GSRT**			**Similarity, Right Triangles, & Trigonometry**	**1-4**	**3-11%**
10	GSRT	A	0	**Understand similarity in terms of similarity transformations.**	**0-2**	**0-6%**
10	GSRT	A	1	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	0-2	0-6%

10	GSRT	A	2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	0-2	0-6%
10	GSRT	A	3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	0-2	0-6%
10	GSRT	B	0	**Prove theorems involving similarity.**	**0-2**	**0-6%**
10	GSRT	B	4	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	0-2	0-6%
10	GSRT	B	5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	0-2	0-6%
10	GSRT	C	0	**Define trigonometric ratios and solve problems involving right triangles.**	**1-3**	**3-9%**
10	GSRT	C	6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	0-2	0-6%
10	GSRT	C	7	Explain and use the relationship between the sine and cosine of complementary angles.	0-2	0-6%
10	GSRT	C	8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	1-2	3-6%
10	**GC**			**Circles**	**0-2**	**0-6%**
10	GC	A	0	**Understand and apply theorems about circles.**	**0-2**	**0-6%**
10	GC	A	1	Prove that all circles are similar.	0-1	0-3%
10	GC	A	2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	0-1	0-3%
10	GC	A	3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	0-1	0-3%
10	GC	B	0	**Find arc lengths and areas of sectors of circles.**	**0-1**	**0-3%**

10	GC	B	5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	0-1	0-3%
10	**GGPE**			**Expressing Geometric Properties with Equations**	**1-5**	**3-14%**
10	GGPE	A	0	**Translate between the geometric description and the equation for a conic section.**	**0-2**	**0-6%**
10	GGPE	A	1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	0-2	0-6%
10	GGPE	A	2	Derive the equation of a parabola given a focus and directrix.	0-1	0-3%
10	GGPE	B	0	**Use coordinates to prove simple geometric theorems algebraically.**	**0-4**	**0-11%**
10	GGPE	B	4	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,sqt3) lies on the circle centered at the origin and containing the point (0, 2).	0-2	0-6%
10	GGPE	B	5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	0-2	0-6%
10	GGPE	B	6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	0-2	0-6%
10	GGPE	B	7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	0-2	0-6%
10	**GGMD**			**Geometric Measurement & Dimension**	**1-3**	**3-9%**
10	GGMD	A	0	**Explain volume formulas and use them to solve problems.**	**0-3**	**0-9%**
10	GGMD	A	1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	0-2	0-6%
10	GGMD	A	3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	0-2	0-6%

10	GGMD	B	0	**Visualize relationships between two-dimensional and three-dimensional objects.**	**0-2**	**0-6%**
10	GGMD	B	4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	0-1	0-3%
10	GGMD	B	IA7	Plot points in three-dimensions.	0-1	0-3%
10	**GMG**			**Modeling with Geometry**	**0-2**	**0-6%**
10	GMG	A	0	**Apply geometric concepts in modeling situations.**	**0-2**	**0-6%**
10	GMG	A	1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	0-2	0-6%
10	GMG	A	2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	0-1	0-3%
10	GMG	A	3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	0-1	0-3%
10	GMG	B	0	**Use diagrams consisting of vertices and edges (vertex-edge graphs) to model and solve problems related to networks.**	**0-1**	**0-3%**
10	GMG	B	IA8	Understand, analyze, evaluate, and apply vertex-edge graphs to model and solve problems related to paths, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings.	0-1	0-3%
10	GMG	B	IA9	Model and solve problems using at least two of the following fundamental graph topics and models: Euler paths and circuits, Hamilton paths and circuits, the traveling salesman problem (TSP), minimum spanning trees, critical paths, vertex coloring.	0-1	0-3%
10	GMG	B	IA10	Compare and contrast vertex-edge graph topics and models in terms of: properties algorithms optimization types of problems that can be solved	0-1	0-3%
10	**S**			**Statistics & Probability**	**4-6**	**11-17%**
10	**SID**			**Interpreting Categorical & Quantitative Data**	**1-5**	**3-14%**

10	SID	A	0	**Summarize, represent, and interpret data on a single count or measurement variable.**	**0-3**	**0-9%**
10	SID	A	1	Represent data with plots on the real number line (dot plots, histograms, and box plots).	0-3	0-9%
10	SID	A	2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	0-2	0-6%
10	SID	A	3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	0-2	0-6%
10	SID	B	0	**Summarize, represent, and interpret data on two categorical and quantitative variables.**	**0-3**	**0-9%**
10	SID	B	5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	0-2	0-6%
10	SID	B	6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	0-2	0-6%
10	SID	C	0	**Interpret linear models.**	**0-3**	**0-9%**
10	SID	C	7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	0-2	0-6%
10	SID	C	8	Compute (using technology) and interpret the correlation coefficient of a linear fit.	0-1	0-3%
10	SID	C	9	Distinguish between correlation and causation.	0-1	0-3%
10	**SCP**			**Conditional Probability & the Rules of Probability**	**0-4**	**0-11%**
10	SCP	A	0	**Understand independence and conditional probability and use them to interpret data.**	**0-4**	**0-11%**

10	SCP	A	1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	0-3	0-9%
10	SCP	A	2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	0-2	0-6%
10	SCP	A	3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	0-2	0-6%
10	SCP	A	4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.	0-2	0-6%
10	SCP	A	5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.	0-2	0-6%
10	SCP	B	0	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**	**0-2**	**0-6%**
10	SCP	B	6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	0-1	0-3%
10	SCP	B	7	Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.	0-1	0-3%

Grade 11

Grade	Domain	Cluster	Standard #	Description	Range of operational items	Approx % of total test
11	**N**			**Number and Quantity**	**6-8**	**17-23%**
11	**NRN**			**The Real Number System**	**1-5**	**3-14%**
11	NRN	A	0	**Extend the properties of exponents to rational exponents.**	**1-4**	**3-11%**
11	NRN	A	1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 5^{1/3} to be the cube root of 5 because we want (5^{1/3})^3 = 5^{(1/3)3} to hold, so (5^{1/3})^3 must equal 5.	0-3	0-9%
11	NRN	A	2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	0-4	0-11%
11	NRN	B	0	**Use properties of rational and irrational numbers.**	**0-3**	**0-9%**
11	NRN	B	3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.	0-3	0-9%
11	**NQ**			**Quantities**	**1-5**	**3-14%**
11	NQ	A	0	**Reason quantitatively and use units to solve problems.**	**1-5**	**3-14%**
11	NQ	A	1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	1-3	3-9%
11	NQ	A	2	Define appropriate quantities for the purpose of descriptive modeling.	0-3	0-9%
11	NQ	A	3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	0-3	0-9%
11	NQ	B	0	**Understand and apply the mathematics of voting.**	**0-1**	**0-3%**

11	NQ	B	IA3	Understand, analyze, apply, and evaluate some common voting and analysis methods in addition to majority and plurality, such as runoff, approval, the so-called instant-runoff voting (IRV) method, the Borda method and the Condorcet method.	0-1	0-3%
11	NQ	C	0	**Understand and apply some basic mathematics of information processing and the Internet.**	**0-1**	**0-3%**
11	NQ	C	IA4	Describe the role of mathematics in information processing, particularly with respect to the Internet.	0-1	0-3%
11	NQ	C	IA5	Understand and apply elementary set theory and logic as used in simple Internet searches.	0-1	0-3%
11	NQ	C	IA6	Understand and apply basic number theory, including modular arithmetic, for example, as used in keeping information secure through public-key cryptography.	0-1	0-3%
11	**NCN**			**The Complex Number System**	**0-5**	**0-14%**
11	NCN	A	0	**Perform arithmetic operations with complex numbers.**	**0-4**	**0-11%**
11	NCN	A	1	Know there is a complex number i such that i^2 = -1, and every complex number has the form a + bi with a and b real.	0-1	0-3%
11	NCN	A	2	Use the relation i^2 = -1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	0-3	0-9%
11	NCN	C	0	**Use complex numbers in polynomial identities and equations.**	**0-2**	**0-6%**
11	NCN	C	7	Solve quadratic equations with real coefficients that have complex solutions.	0-1	0-3%
11	NCN	C	8	(+) Extend polynomial identities to the complex numbers. For example, rewrite x^2 + 4 as (x + 2i)(x - 2i).	0-1	0-3%
11	NCN	C	9	(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	0-1	0-3%
11	**NVM**			**Vector & Matrix Quantities**	**0-2**	**0-6%**
11	NVM	A	0	**Represent and model with vector quantities**	**0-2**	**0-6%**
11	NVM	A	1	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).	0-1	0-3%
11	NVM	A	2	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	0-1	0-3%

11	NVM	A	3	(+) Solve problems involving velocity and other quantities that can be represented by vectors.	0-1	0-3%
11	NVM	B	0	**Perform operations on vectors.**	**0-2**	**0-6%**
11	NVM	B	4	(+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	0-1	0-3%
11	NVM	B	5	(+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(v{sub x}, v{sub y}) = (cv{sub x}, cv{sub y}). b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ≠ 0, the direction of cv is either along v (for c > 0) or against v (for c < 0).	0-1	0-3%
11	NVM	C	0	**Perform operations on matrices and use matrices in applications.**	**0-2**	**0-6%**
11	NVM	C	6	(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.	0-1	0-3%
11	NVM	C	7	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.	0-1	0-3%
11	NVM	C	8	(+) Add, subtract, and multiply matrices of appropriate dimensions.	0-1	0-3%
11	NVM	C	9	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.	0-1	0-3%
11	**F**			**Functions**	**7-9**	**20-26%**
11	**FIF**			**Interpreting Functions**	**1-5**	**3-14%**
11	FIF	A	0	**Understand the concept of a function and use function notation.**	**0-3**	**0-9%**

11	FIF	A	1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).	0-2	0-6%
11	FIF	A	2	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.	0-2	0-6%
11	FIF	A	3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n + 1) = f(n) + f(n - 1) for n ≥ 1.	0-1	0-3%
11	FIF	B	0	**Interpret functions that arise in applications in terms of the context.**	**0-3**	**0-9%**
11	FIF	B	4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.	0-2	0-6%
11	FIF	B	5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.	0-1	0-3%
11	FIF	B	6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.	0-2	0-6%
11	FIF	C	0	**Analyze functions using different representations.**	**0-3**	**0-9%**

11	FIF	C	7abce	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.	0-3	0-9%
11	FIF	C	7d	d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.	0-1	0-3%
11	FIF	C	8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)^t, y = (0.97)^t, y = (1.01)^{12t}, y = (1.2)^{t/10}, and classify them as representing exponential growth or decay.	0-2	0-6%
11	FIF	C	9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.	0-2	0-6%
11	**FBF**			**Building Functions**	**1-3**	**3-9%**
11	FBF	A	0	**Build a function that models a relationship between two quantities.**	**0-3**	**0-9%**
11	FBF	A	1ab	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.	0-2	0-6%

11	FBF	A	1c	c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.	0-1	0-3%
11	FBF	A	2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	0-2	0-6%
11	FBF	B	0	**Build new functions from existing functions.**	**0-2**	**0-6%**
11	FBF	B	3	Identify the effect on the graph of replacing f(x) by f(x) + k, kf(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.	0-2	0-6%
11	FBF	B	4a	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) = 2x^3 or f(x) = (x + 1)/(x - 1) for x ≠ 1.	0-1	0-3%
11	FBF	B	4bcd	b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.	0-1	0-3%
11	**FLE**			**Linear, Quadratic, & Exponential Models**	**1-4**	**3-11%**
11	FLE	A	0	**Construct and compare linear, quadratic, and exponential models and solve problems.**	**0-4**	**0-11%**
11	FLE	A	1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	0-2	0-6%
11	FLE	A	2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	0-2	0-6%

11	FLE	A	3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.	0-2	0-6%
11	FLE	A	4	For exponential models, express as a logarithm the solution to ab^{ct} = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.	0-2	0-6%
11	FLE	B	0	**Interpret expressions for functions in terms of the situation they model.**	**0-1**	**0-3%**
11	FLE	B	5	Interpret the parameters in a linear or exponential function in terms of a context.	0-1	0-3%
11	**FTF**			**Trigonometric Functions**	**0-2**	**0-6%**
11	FTF	A	0	**Extend the domain of trigonometric functions using the unit circle.**	**0-1**	**0-3%**
11	FTF	A	1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	0-1	0-3%
11	FTF	A	2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	0-1	0-3%
11	FTF	A	3	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for pi/3, pi/4 and pi/6, and use the unit circle to express the values of sine, cosine, and tangent for x, pi + x, and 2pi - x in terms of their values for x, where x is any real number.	0-1	0-3%
11	FTF	B	0	**Model periodic phenomena with trigonometric functions.**	**0-1**	**0-3%**
11	FTF	B	5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	0-1	0-3%
11	FTF	C	0	**Prove and apply trigonometric identities.**	**0-1**	**0-3%**
11	FTF	C	8	Prove the Pythagorean identity sin^2(theta) + cos^2(theta) = 1 and use it to calculate trigonometric ratios.	0-1	0-3%
11	**A**			**Algebra**	**7-9**	**20-26%**
11	**ASSE**			**Seeing Structure in Expressions**	**1-3**	**3-9%**
11	ASSE	A	0	**Interpret the structure of expressions.**	**0-2**	**0-6%**

11	ASSE	A	1	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)^n as the product of P and a factor not depending on P.	0-2	0-6%
11	ASSE	A	2	Use the structure of an expression to identify ways to rewrite it. For example, see x^4 - y^4 as (x^2)^2 - (y^2)^2, thus recognizing it as a difference of squares that can be factored as (x^2 - y^2)(x^2 + y^2).	0-2	0-6%
11	ASSE	B	0	**Write expressions in equivalent forms to solve problems.**	**0-2**	**0-6%**
11	ASSE	B	3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.15^{1/12})^{12t} ≈ 1.012^{12t} to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.	0-2	0-6%
11	ASSE	B	4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.	0-1	0-3%
11	**AAPR**			**Arithmetic with Polynomials & Rational Expressions**	**1-4**	**3-11%**
11	AAPR	A	0	**Perform arithmetic operations on polynomials.**	**0-3**	**0-9%**
11	AAPR	A	1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.	0-3	0-9%
11	AAPR	B	0	**Understand the relationship between zeros and factors of polynomials.**	**0-2**	**0-6%**
11	AAPR	B	2	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x - a) is a factor of p(x).	0-1	0-3%
11	AAPR	B	3	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	0-1	0-3%

11	AAPR	C	0	**Use polynomial identities to solve problems.**	**0-1**	**0-3%**
11	AAPR	C	4	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x^2 + y^2)^2 = (x^2 + y^2)^2 + (2xy)^2 can be used to generate Pythagorean triples.	0-1	0-3%
11	AAPR	D	0	**Rewrite rational expressions.**	**0-2**	**0-6%**
11	AAPR	D	6	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.	0-1	0-3%
11	AAPR	D	7	(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.	0-1	0-3%
11	**ACED**			**Creating Equations**	**1-4**	**3-11%**
11	ACED	A	0	**Create equations that describe numbers or relationships.**	**1-4**	**3-11%**
11	ACED	A	1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	0-2	0-6%
11	ACED	A	2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	0-2	0-6%
11	ACED	A	3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.	0-2	0-6%
11	ACED	A	4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	0-2	0-6%
11	**AREI**			**Reasoning with Equations & Inequalities**	**1-4**	**3-11%**
11	AREI	A	0	**Understand solving equations as a process of reasoning and explain the reasoning.**	**0-2**	**0-6%**

11	AREI	A	1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.	0-2	0-6%
11	AREI	A	2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.	0-2	0-6%
11	AREI	B	0	**Solve equations and inequalities in one variable.**	**1-3**	**3-9%**
11	AREI	B	3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.	0-2	0-6%
11	AREI	B	4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x - p)^2 = q that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for x^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.	0-2	0-6%
11	AREI	C	0	**Solve systems of equations**	**0-2**	**0-6%**
11	AREI	C	5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	0-1	0-3%
11	AREI	C	6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.	0-1	0-3%
11	AREI	C	7	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = -3x and the circle x^2 + y^2 = 3.	0-1	0-3%
11	AREI	D	0	**Represent and solve equations and inequalities graphically.**	**0-2**	**0-6%**
11	AREI	D	10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).	0-1	0-3%

11	AREI	D	11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.	0-1	0-3%
11	AREI	D	12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	0-1	0-3%
11	**G**			**Geometry**	**6-8**	**17-23%**
11	**GCO**			**Congruence**	**1-4**	**3-11%**
11	GCO	A	0	**Experiment with transformations in the plane.**	**0-3**	**0-9%**
11	GCO	A	1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	0-2	0-6%
11	GCO	A	2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).	0-2	0-6%
11	GCO	A	3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.	0-2	0-6%
11	GCO	A	4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	0-2	0-6%
11	GCO	A	5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.	0-2	0-6%
11	GCO	B	0	**Understand congruence in terms of rigid motions.**	**0-2**	**0-6%**

11	GCO	B	6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.	0-2	0-6%
11	GCO	B	7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.	0-2	0-6%
11	GCO	B	8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.	0-2	0-6%
11	GCO	C	0	**Prove geometric theorems.**	**0-3**	**0-9%**
11	GCO	C	9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.	0-2	0-6%
11	GCO	C	10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.	0-2	0-6%
11	GCO	C	11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.	0-2	0-6%
11	GCO	D	0	**Make geometric constructions.**	**0-2**	**0-6%**
11	GCO	D	12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.	0-2	0-6%
11	GCO	D	13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.	0-1	0-3%
11	**GSRT**			**Similarity, Right Triangles, & Trigonometry**	**1-4**	**3-11%**

11	GSRT	A	0	**Understand similarity in terms of similarity transformations.**	**0-2**	**0-6%**
11	GSRT	A	1	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	0-2	0-6%
11	GSRT	A	2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	0-2	0-6%
11	GSRT	A	3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	0-2	0-6%
11	GSRT	B	0	**Prove theorems involving similarity.**	**0-2**	**0-6%**
11	GSRT	B	4	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	0-2	0-6%
11	GSRT	B	5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	0-2	0-6%
11	GSRT	C	0	**Define trigonometric ratios and solve problems involving right triangles.**	**0-3**	**0-9%**
11	GSRT	C	6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	0-2	0-6%
11	GSRT	C	7	Explain and use the relationship between the sine and cosine of complementary angles.	0-2	0-6%
11	GSRT	C	8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	0-2	0-6%
11	GSRT	D	0	**Apply trigonometry to general triangles.**	**0-2**	**0-6%**
11	GSRT	D	10	(+) Prove the Laws of Sines and Cosines and use them to solve problems.	0-1	0-3%
11	GSRT	D	11	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).	0-1	0-3%

11	**GC**			**Circles**	**0-2**	**0-6%**
11	GC	A	0	**Understand and apply theorems about circles.**	**0-2**	**0-6%**
11	GC	A	1	Prove that all circles are similar.	0-1	0-3%
11	GC	A	2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	0-1	0-3%
11	GC	A	3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	0-1	0-3%
11	GC	B	0	**Find arc lengths and areas of sectors of circles.**	**0-1**	**0-3%**
11	GC	B	5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.	0-1	0-3%
11	**GGPE**			**Expressing Geometric Properties with Equations**	**1-5**	**3-14%**
11	GGPE	A	0	**Translate between the geometric description and the equation for a conic section.**	**0-2**	**0-6%**
11	GGPE	A	1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	0-2	0-6%
11	GGPE	A	2	Derive the equation of a parabola given a focus and directrix.	0-1	0-3%
11	GGPE	A	3	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.	0-1	0-3%
11	GGPE	B	0	**Use coordinates to prove simple geometric theorems algebraically.**	**0-4**	**0-11%**
11	GGPE	B	4	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1,sqt3) lies on the circle centered at the origin and containing the point (0, 2).	0-2	0-6%

11	GGPE	B	5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).	0-2	0-6%
11	GGPE	B	6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	0-2	0-6%
11	GGPE	B	7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.	0-2	0-6%
11	**GGMD**			**Geometric Measurement & Dimension**	**0-3**	**0-9%**
11	GGMD	A	0	**Explain volume formulas and use them to solve problems.**	**0-3**	**0-9%**
11	GGMD	A	1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.	0-2	0-6%
10	GGMD	A	2	(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.	0	0%
11	GGMD	A	3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.	0-2	0-6%
11	GGMD	B	0	**Visualize relationships between two-dimensional and three-dimensional objects.**	**0-2**	**0-6%**
11	GGMD	B	4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	0-1	0-3%
11	GGMD	B	IA7	Plot points in three-dimensions.	0-1	0-3%
11	**GMG**			**Modeling with Geometry**	**0-2**	**0-6%**
11	GMG	A	0	**Apply geometric concepts in modeling situations.**	**0-2**	**0-6%**
11	GMG	A	1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	0-2	0-6%
11	GMG	A	2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	0-1	0-3%
11	GMG	A	3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	0-1	0-3%

11	GMG	B	0	**Use diagrams consisting of vertices and edges (vertex-edge graphs) to model and solve problems related to networks.**	**0-1**	**0-3%**
11	GMG	B	IA8	Understand, analyze, evaluate, and apply vertex-edge graphs to model and solve problems related to paths, circuits, networks, and relationships among a finite number of elements, in real-world and abstract settings.	0-1	0-3%
11	GMG	B	IA9	Model and solve problems using at least two of the following fundamental graph topics and models: Euler paths and circuits, Hamilton paths and circuits, the traveling salesman problem (TSP), minimum spanning trees, critical paths, vertex coloring.	0-1	0-3%
11	GMG	B	IA10	Compare and contrast vertex-edge graph topics and models in terms of: properties algorithms optimization types of problems that can be solved	0-1	0-3%
11	**S**			**Statistics & Probability**	**4-6**	**11-17%**
11	**SID**			**Interpreting Categorical & Quantitative Data**	**1-4**	**3-11%**
11	SID	A	0	**Summarize, represent, and interpret data on a single count or measurement variable.**	**0-3**	**0-9%**
11	SID	A	1	Represent data with plots on the real number line (dot plots, histograms, and box plots).	0-3	0-9%
11	SID	A	2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.	0-2	0-6%
11	SID	A	3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).	0-2	0-6%
11	SID	A	4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	0-1	0-3%
11	SID	B	0	**Summarize, represent, and interpret data on two categorical and quantitative variables.**	**0-3**	**0-9%**

11	SID	B	5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.	0-2	0-6%
11	SID	B	6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.	0-2	0-6%
11	SID	C	0	**Interpret linear models.**	**0-3**	**0-9%**
11	SID	C	7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.	0-2	0-6%
11	SID	C	8	Compute (using technology) and interpret the correlation coefficient of a linear fit.	0-1	0-3%
11	SID	C	9	Distinguish between correlation and causation.	0-1	0-3%
11	**SIC**			**Making Inferences & Justifying Conclusions**	**0-3**	**0-9%**
11	SIC	A	0	**Understand and evaluate random processes underlying statistical experiments.**	**0-2**	**0-6%**
11	SIC	A	1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.	0-1	0-3%
11	SIC	A	2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.Would a result of 5 tails in a row cause you to question the model?	0-1	0-3%
11	SIC	B	0	**Make inferences and justify conclusions from sample surveys, experiments, and observational studies.**	**0-3**	**0-9%**
11	SIC	B	3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.	0-1	0-3%

11	SIC	B	4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.	0-1	0-3%
11	SIC	B	5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.	0-1	0-3%
11	SIC	B	6	Evaluate reports based on data.	0-1	0-3%
11	**SCP**			**Conditional Probability & the Rules of Probability**	**1-4**	**3-11%**
11	SCP	A	0	**Understand independence and conditional probability and use them to interpret data.**	**0-4**	**0-11%**
11	SCP	A	1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").	0-3	0-9%
11	SCP	A	2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	0-2	0-6%
11	SCP	A	3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	0-2	0-6%
11	SCP	A	4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.	0-2	0-6%
11	SCP	A	5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.	0-2	0-6%

11	SCP	B	0	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**	**0-3**	**0-9%**
11	SCP	B	6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	0-1	0-3%
11	SCP	B	7	Apply the Addition Rule, P(A or B) = P(A) + P(B) - P(A and B), and interpret the answer in terms of the model.	0-1	0-3%
11	SCP	B	8	(+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	0-1	0-3%
11	SCP	B	9	(+) Use permutations and combinations to compute probabilities of compound events and solve problems.	0-1	0-3%

References

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Appendix: Form Alignments to Test Blueprints, 2019-2022

Table 14. ISASP Mathematics Grade 3 Blueprint Alignments, 2019 -2022 administrations

Iowa Core Mathematics Domains	Number of items per blueprint (2019-2022)	Number of items at partially/fully aligned (2019)	Blueprint met	Number of items at partially/fully aligned (2021)	Blueprint met	Number of items at partially/fully aligned (2022)	Blueprint met
Operations and Algebraic Thinking	11-13	12	Yes	11	Yes	11	Yes
Number and Operations in Base Ten	5-7	6	Yes	6	Yes	6	Yes
Number and Operations – Fractions	4-5	4	Yes	4	Yes	4	Yes
Measurement and Data	8-10	9	Yes	10	Yes	10	Yes
Geometry	4-5	3	No	4	Yes	4	Yes

Table 15. ISASP Mathematics Grade 4 Blueprint Alignments, 2019 -2022 administrations

Iowa Core Mathematics Domains	Number of items per blueprint (2019-2022)	Number of items at partially/fully aligned (2019)	Blueprint met	Number of items at partially/fully aligned (2021)	Blueprint met	Number of items at partially/fully aligned (2022)	Blueprint met
Operations and Algebraic Thinking	5-7	6	Yes	6	Yes	6	Yes
Number and Operations in Base Ten	9-11	10	Yes	10	Yes	10	Yes
Number and Operations – Fractions	9-11	10	Yes	10	Yes	10	Yes
Measurement and Data	7-9	8	Yes	8	Yes	8	Yes
Geometry	5-7	6	Yes	6	Yes	6	Yes

Table 16. ISASP Mathematics Grade 5 Blueprint Alignments, 2019 -2022 administrations

Iowa Core Mathematics Domains	Number of items per blueprint (2019-2022)	Number of items at partially/fully aligned (2019)	Blueprint met	Number of items at partially/fully aligned (2021)	Blueprint met	Number of items at partially/fully aligned (2022)	Blueprint met
Operations and Algebraic Thinking	6-8	7	Yes	7	Yes	7	Yes
Number and Operations in Base Ten	7-9	8	Yes	8	Yes	8	Yes
Number and Operations – Fractions	8-10	9	Yes	9	Yes	9	Yes
Measurement and Data	7-9	8	Yes	8	Yes	8	Yes
Geometry	4-6	5	Yes	5	Yes	5	Yes

Table 17. ISASP Mathematics Grade 6 Blueprint Alignments, 2019 -2022 administrations

Iowa Core Mathematics Domains	Number of items per blueprint (2019-2022)	Number of items at partially/fully aligned (2019)	Blueprint met	Number of items at partially/fully aligned (2021)	Blueprint met	Number of items at partially/fully aligned (2022)	Blueprint met
Geometry	8-10	9	Yes	8	Yes	8	Yes
Ratios and Proportional Relationships	8-10	9	Yes	9	Yes	9	Yes
The Number System	9-11	10	Yes	10	Yes	10	Yes
Expressions and Equations	10-12	11	Yes	11	Yes	11	Yes
Statistics and Probability	5-7	6	Yes	7	Yes	7	Yes

Table 18. ISASP Mathematics Grade 7 Blueprint Alignments, 2019 -2022 administrations

Iowa Core Mathematics Domains	Number of items per blueprint (2019-2022)	Number of items at partially/fully aligned (2019)	Blueprint met	Number of items at partially/fully aligned (2021)	Blueprint met	Number of items at partially/fully aligned (2022)	Blueprint met
Geometry	5-7	6	Yes	6	Yes	6	Yes
Ratios and Proportional Relationships	6-8	7	Yes	8	Yes	8	Yes
The Number System	9-11	10	Yes	10	Yes	10	Yes
Expressions and Equations	12-14	13	Yes	13	Yes	13	Yes
Statistics and Probability	5-7	6	Yes	5	Yes	5	Yes

Table 19. ISASP Mathematics Grade 8 Blueprint Alignments, 2019 -2022 administrations

Iowa Core Mathematics Domains	Number of items per blueprint (2019-2022)	Number of items at partially/fully aligned (2019)	Blueprint met	Number of items at partially/fully aligned (2021)	Blueprint met	Number of items at partially/fully aligned (2022)	Blueprint met
Geometry	4-6	5	Yes	5	Yes	5	Yes
Statistics and Probability	4-6	4	Yes	5	Yes	5	Yes
Functions	6-8	6	Yes	7	Yes	7	Yes
Algebra	10-12	11	Yes	11	Yes	11	Yes
Number and Quantity	6-8	7	Yes	7	Yes	7	Yes

Table 20. ISASP Mathematics Grade 9 Blueprint Alignments, 2019 -2022 administrations

Iowa Core Mathematics Domains	Number of items per blueprint (2019-2022)	Number of items at partially/fully aligned (2019)	Blueprint met	Number of items at partially/fully aligned (2021)	Blueprint met	Number of items at partially/fully aligned (2022)	Blueprint met
Geometry	9-11	10	Yes	9	Yes	9	Yes
The Number System	4-6	5	Yes	5	Yes	5	Yes
Expressions and Equations	13-15	13	Yes	15	Yes	15	Yes
Statistics and Probability	7-9	8	Yes	9	Yes	9	Yes
Functions	9-11	10	Yes	9	Yes	9	Yes

Table 21. ISASP Mathematics Grade 10 Blueprint Alignments, 2019 -2022 administrations

Iowa Core Mathematics Domains	Number of items per blueprint (2019-2022)	Number of items at partially/fully aligned (2019)	Blueprint met	Number of items at partially/fully aligned (2021)	Blueprint met	Number of items at partially/fully aligned (2022)	Blueprint met
Geometry	6-8	7	Yes	8	Yes	8	Yes
Statistics and Probability	4-6	5	Yes	5	Yes	5	Yes
Functions	7-9	7	Yes	7	Yes	7	Yes
Algebra	7-9	8	Yes	8	Yes	8	Yes
Number and Quantity	6-8	7	Yes	7	Yes	7	Yes

Table 22. ISASP Mathematics Grade 11 Blueprint Alignments, 2019 -2022 administrations

Iowa Core Mathematics Domains	Number of items per blueprint (2019-2022)	Number of items at partially/fully aligned (2019)	Blueprint met	Number of items at partially/fully aligned (2021)	Blueprint met	Number of items at partially/fully aligned (2022)	Blueprint met
Geometry	9-11	10	Yes	10	Yes	10	Yes
Statistics and Probability	4-6	5	Yes	6	Yes	6	Yes
Functions	5-7	6	Yes	5	Yes	5	Yes
Algebra	6-8	7	Yes	7	Yes	7	Yes
Number and Quantity	6-8	7	Yes	7	Yes	7	Yes